Chapter 9

Turbulent Jets

SUMMARY: This chapter is concerned with turbulent jets, namely their overall shape and velocity structure. The first jets being considered are those penetrating in homogeneous fluids, and the theory is later extended to consider the effects of a cross-current and of ambient buoyancy. Puffs, which are intermittent injections of momentum, are briefly considered.

9.1 Intrusion of a fluid into another

In environmental fluids, it is not a rare occurrence to see one fluid intruding into another. Common examples are wastewater discharges from pipes into rivers or lakes and plumes exiting from industrial smokestacks. In every case, a fluid with some momentum and/or buoyancy exits from a relatively narrow orifice and intrudes into a larger body of fluid with different characteristics, such as different speed, temperature or contamination level.

It is helpful to categorize the various types of intrusion according to whether they inject momentum, buoyancy or both in the ambient fluid, and whether or not they persist in time (Table 9.1).

Table 9.1 Types of intrusions of a fluid into another and the corresponding terminology.

	Continuous injection	Intermittent injection
Momentum only	Jet	Puff
Buoyancy only	Plume	Thermal
Both momentum and buoyancy	Buoyant jet or forced plume	Buoyant puff

These flows can be characterized as partly turbulent because they create situations where the turbulence level is much higher in the vicinity of the intrusion than in the surrounding fluid. The present chapter is devoted to momentum-only sources, *i.e.* jets and puffs.

9.2 Turbulent Jets

Whenever a moving fluid enters a quiescent body of the same fluid, a velocity shear is created between the entering and ambient fluids, causing turbulence and mixing. In nature, the situation occurs where a river empties in a lake or estuary, or occasionally when a wind blows through an orographic gap. But, perhaps the most clearly defined jets are those produced when a fluid is discharged in the environment through a relatively narrow conduit, such as an industrial discharge released through a pipe on the bank of a river, lake, or coastal ocean.

Since the properties of a turbulent flow greatly depend on the geometry of the flow domain and on the type of forces acting on the fluid, almost every situation is a separate problem requiring specific investigation. We shall therefore limit ourselves here to the most basic case, that of a jet penetrating in an otherwise quiescent fluid.

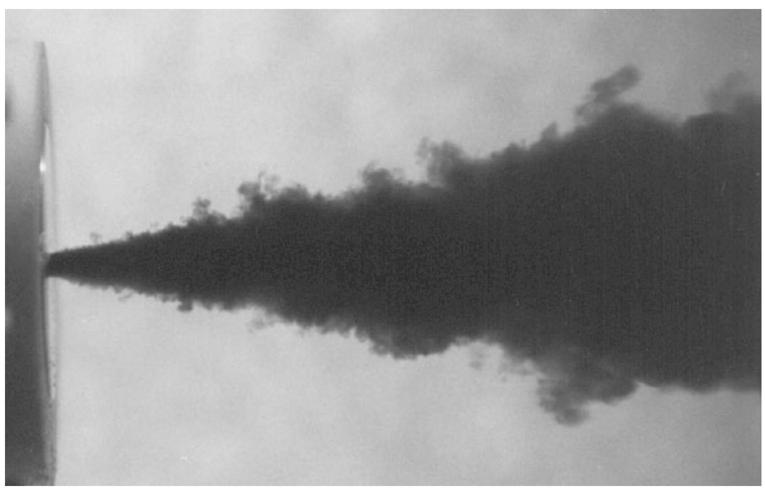

Figure 9.1: A water jet emerging from a nozzle into an otherwise undisturbed tank of water. The jet water is colored to be made visible.

Laboratory investigations of jets penetrating into a quiescent fluid of the same density (*e.g.*, Figure 9.1) consistently reveal that the envelope containing the turbulence caused by the jet adopts a nearly conical shape. In other words, the radius R of the jet is proportional to the distance x downstream from the discharge location. Further, the opening angle is always the same, regardless of the nature of the fluid (air or water) and of other circumstances (such as diameter of outlet and discharge speed). This universal angle is 11.8° giving approximately 24° from side to opposite side. It follows that the coefficient of proportionality between the jet radius R and the downstream distance x from the outlet is $\tan(11.8°) \simeq 1/5$:

$$R(x) \;=\; \frac{1}{5}\,x. \tag{9.1}$$

Note that since the initial jet radius is not zero but the finite nozzle radius, equal to half the exit diameter d, the distance x must be counted not from the orifice but from a distance $5d/2$ into the conduit. This point of origin is called the *virtual source* (Figure 9.2).

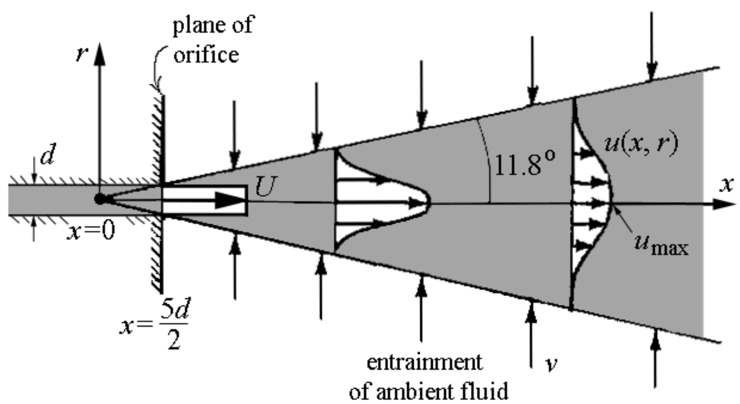

Figure 9.2: Schematic description of a jet penetrating in a fluid at rest. The widening is linear with distance, and all cross-jet velocity profiles, except those very near the orifice, are similar to one another, after suitable averaging over turbulent fluctuations.

Observations suitably averaged over the many turbulent fluctuations (Figures 9.3 and 9.4) reveal that the velocity in the jet obeys a law of similarity: All cross-sections appear identical, except for a stretching factor, and the velocity profile across the jet exhibits a nearly Gaussian shape (bell curve). Therefore, we can write:

$$u(x,r) \;=\; u_{\max}\;\exp\left(-\,\frac{r^2}{2\sigma^2}\right) \tag{9.2}$$

where x is the downstream distance along the jet (counted from the virtual source), r is the cross-jet radial distance from its centerline, $u_{\max}(x)$ is the maximum speed at the centerline, and $\sigma(x)$ is the standard deviation related to the spread of the profile across the centerline. Since 4σ is the width of the distribution that encompasses 95% of the area under the curve (a traditional and practical measure borrowed

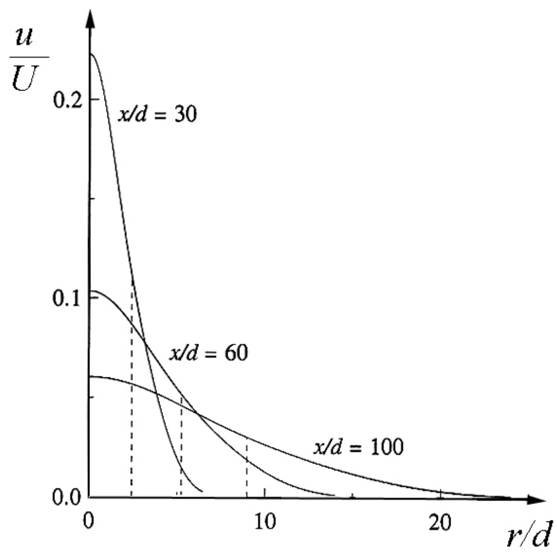

Figure 9.3: Radial profiles of mean axial velocity in a turbulent round jet at Reynolds number $Re = 95500$. The dashed lines indicate the half-width, $r_{50\%}$, of each profile. (Adapted from Pope, 2000)

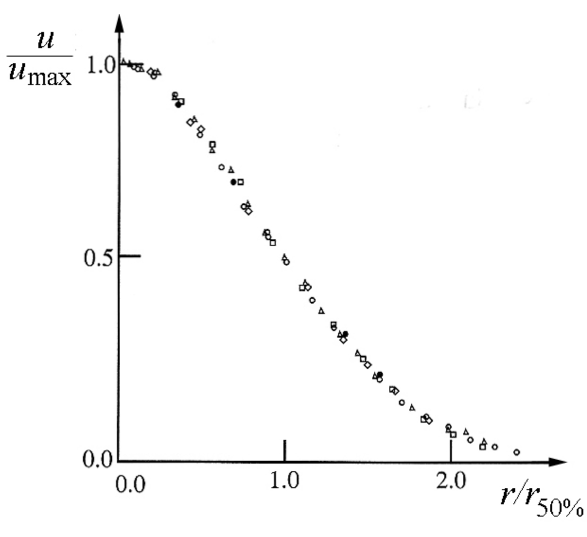

Figure 9.4: Mean axial velocity versus radial distance in a turbulent round jet at Reynolds number $Re \approx 10^5$. The velocity is scaled by the maximum value at the center of the jet, and the radial distance by $r_{50\%}$, the distance at which the velocity drops to half of its maximum value. (Adapted from Pope, 2000)

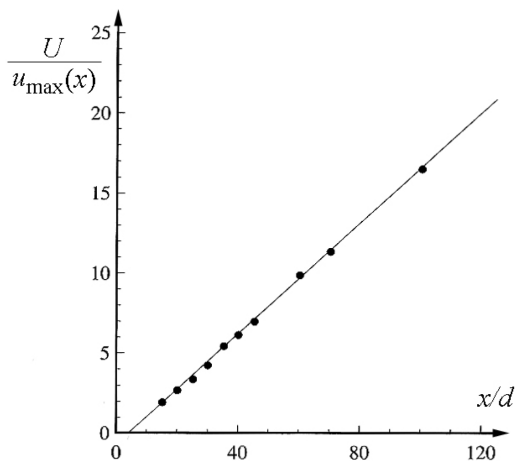

Figure 9.5: The variation of the maximum velocity in a round jet versus distance along the axis of the jet. (Adapted from Pope, 2000)

from statistics) and since we know it to be the diameter $2R$ of the jet, we can write $4\sigma = 2R$, *i.e.* $\sigma = x/10$, which leads to:

$$u(x,r) = u_{\max} \exp\left(-\frac{50r^2}{x^2}\right) . \tag{9.3}$$

When a jet enters a fluid at rest, the sole source of momentum is that of the jet itself, and the absence of external accelerating or decelerating forces implies that the momentum flux in the jet's cross-section remains constant downstream. Since this flux is the momentum per unit volume, ρu (where ρ is the fluid density and u the velocity), times the velocity u itself cumulated over the jet's cross-section, the statement that momentum is constant downstream is:

$$\int_0^\infty \rho u^2 \, 2\pi r dr = \rho U^2 \frac{\pi d^2}{4} ,$$

where U and d are respectively the average exit velocity and the orifice diameter, which are usually known. After calculating the integral and by virtue of (9.3), we deduce:

$$u_{\max} = \frac{5d}{x} U. \tag{9.4}$$

In other words, the velocity along the centerline of the jet decreases inversely with distance from the virtual source (*i.e.* the ratio $U/u_{\max}$ increases linearly with distance, as seen in Figure 9.5). To this maximum velocity corresponds an average velocity $\bar{u}$ defined by

$$\bar{u} = \frac{1}{\pi R^2} \int_0^\infty u \, 2\pi r dr = \frac{u_{\max}}{2} = \frac{5d}{2x} U. \tag{9.5}$$

The volumetric flux Q is not constant along the jet because of entrainment of quiescent surrounding fluid. It can be calculated as follows

$$Q \;=\; \int_0^\infty u\; 2\pi r dr \;=\; \frac{\pi}{50}\; u_{\max} x^2 \;=\; \frac{\pi}{10}\; dUx,$$

and is found to increase linearly with distance. The entrainment rate E can be defined as the rate at which the volumetric flux grows with distance, namely

$$E \;=\; \frac{dQ}{dx} \;=\; \frac{\pi dU}{10}\;.$$

From this, we can also introduce an entrainment velocity, which is the radial velocity v necessary to carry this entrainment (Figure 9.2). Volume conservation along a section dx of the jet requires:

$$dQ \;=\; v\; dA,$$

where v is the transverse velocity feeding the entrainment and $dA = 2\pi R dx$ is the lateral area of this section of the jet. Substitution of dA and further substitution of R in terms of the distance x provides:

$$\frac{dQ}{dx} \;=\; 2\pi R v \;=\; \frac{2\pi x v}{5}\;.$$

Equating this with the previous expression for dQ/dx yields the value of the entrainment velocity:

$$v \;=\; \frac{Ud}{4x} \;=\; \frac{u_{\max}}{20} \;=\; 0.10\; \bar{u}. \tag{9.6}$$

It can be shown (see Problem 9-5) that the flux of kinetic energy behaves in the opposite way; it decreases with distance.

The preceding remarks dealt with averaged properties of the jet, its width, mean velocity and entrainment velocity. The velocity fluctuations can, too, be characterized. The strong anisotropy of the jet leads to different statistics in the different directions. Not surprisingly, the largest velocity fluctuations occur in the axial direction. The turbulent velocity in this direction, denoted u_* and defined as the root-mean-square (rms) of the axial-velocity fluctuations squared, is found to vary with both longitudinal distance x and radial distance r. Along the centerline, $u_*/\bar{u} \approx 0.25 - 0.28$ (Pope, 2000, page 105 and Figure 5.7). As one progresses outward from centerline to the flank of the jet, the ratio $u_*/\bar{u}$ increases without bound, revealing that turbulent fluctuations are still active where the jet's average velocity is weak. In the orthogonal directions (radial and azimuthal), the rms velocity is about half that in the axial direction.

When the jet contains a contaminant and the ambient fluid does not, this entrainment naturally causes dilution and the contaminant's concentration decreases downstream. Assuming that the concentration profile across the jet is a Gaussian curve (Figure 9.6) similar to that for the velocity profile, we write:

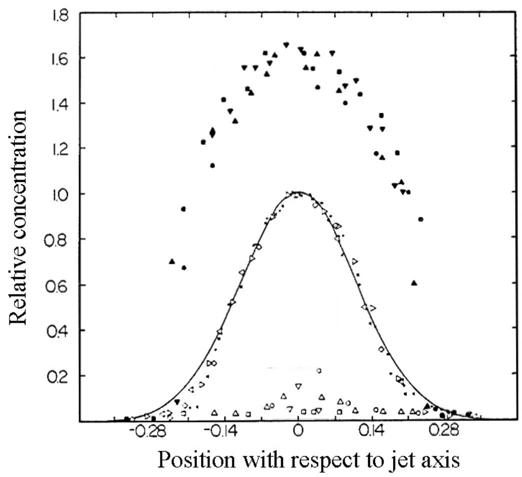

Figure 9.6: Maximum, minimum and time-averaged mean concentration in a planar jet. (Adapted from Kotsovinos, 1975)

$$c(x,r) \;=\; c_{\max}\;\; \exp\left(-\,\frac{r^2}{2\sigma^2}\right) \;=\; c_{\max}\;\; \exp\left(-\,\frac{50r^2}{x^2}\right)\;. \tag{9.7}$$

where $c_{\max}(x)$ is the peak concentration along the centerline, a function of the distance x. Conservation of the total amount of contaminant transported by the jet (assuming that the ambient fluid is free of any contaminant) requires:

$$\int_0^\infty cu\; 2\pi r dr \;=\; c_o U\; \frac{\pi d^2}{4}\;,$$

where c_o is the average concentration at the orifice. Calculation of the integral provides the manner by which the centerline concentration varies along the jet:

$$c_{\max} \;=\; \frac{5d}{x}\; c_o, \tag{9.8}$$

Not surprisingly, because of the dilution generated by the entrainment of ambient fluid, the concentration of the contaminant diminishes with distance from the discharge location. We shall return to this conclusion in our later analysis of smokestack plumes.

9.3 Jets in a Cross-Flow

Text of section

9.4 Puffs

Section 10.6 of Scorer (1997).

9.5 Jets in Stratified Fluids

Text of section

Problems

9-1. An underwater pipe with inner diameter of 25 cm discharges industrial wastewater at the sustained rate of 0.12 m^3/s. The contaminant carried by this discharge increases the density above that of water but, at the same time, the discharge is slightly warmer than the receiving lake water, so that there is no net buoyancy effect. By which distance has the volumetric flowrate quadrupled from its initial value? And, how diluted is the industrial waste at that point? (Define dilution as the ratio of the initial concentration to the local value, to obtain a number larger than unity.)

9-2. A 16-cm underwater pipe discharges into a lake 0.02 m^3/s of wastewater containing 30 mg/L of nitrogen in the form of nitrates. What are the nitrogen concentrations 10, 20 and 50 m away from the pipe's end along the axis of the jet? At which distance has the concentration fallen to 0.5 mg/L?

9-3. A nozzle or diffuser can be fitted onto the end of the pipe mentioned in the preceding problem to reduce or enlarge its exit diameter. What should that new diameter be to ensure that the concentration of nitrogen falls to 0.5 mg/L at a distance of 15 m downstream of the discharge location?

9-4. For a given volumetric flux Q of discharge by a jet, which discharge creates a more vigorous entrainment and more rapid dilution, one with a narrower pipe diameter and higher exit velocity, or one with a wider pipe diameter and a lower exit velocity?

9-5. Show that in a non-buoyant jet, the local Reynolds number Re based on the local center velocity and local jet diameter is invariant with distance along the jet. What does this imply for the smallest turbulent length scale $d_{\min}$? For simplicity, you may assume that the inertial range spans the entire spectrum of turbulent length scales, from longest to shortest.

9-6. Determine the way the energy flux KE (integral of $\rho u^3/2$ across the jet) behaves as a function of distance x along the jet. Show that it decays with distance and explain in a few words why this must be the case. By which distance has the energy flux dropped to 25% of its upstream value at the exit of the discharge pipe?

9-7. Define the energy dissipation rate ϵ as the loss of kinetic energy experienced by the jet along its axis, per mass of fluid. In other words, with the kinetic-energy flux KE defined as in the preceding problem, the drop $KE(x) - KE(x + dx)$ over a stretch dx of the jet is equal to ϵ times the mass of fluid in that strectch. Determine how fast ϵ decreases with distance along the jet.

9-8.

9-9.