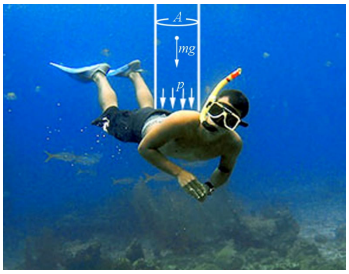
ENGS 34

FLUID MECHANICS

Chapter 2: When the fluid is not moving

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Underwater pressure



Something under water, a person or an object, is subject to the weight of water between it and the water surface.

The weight of the water is mg.

The mass of the water is $m = \rho V$.

The volume of water is $V = A\,h$.

The height h is the distance between the top of the object and the water surface.

The pressure on the object is the weight of water divided by the area A of the object:

$$p = \frac{Weight}{Area} = \frac{\rho\, A\, h\, g}{A} = \rho\, g\, h$$

Then add the atmospheric pressure p_0 of the air onto the water surface:

$$p = \rho\, g\, h + p_0$$

Atmospheric pressure

The atmospheric pressure is the weight of the atmosphere above the object per area exposed.
It obviously decreases with altitude.
At the earth's surface, it varies with the weather.

The standard value is p_0 = 101,325 Pa = 1,013.25 millibars ≈ 1 bar.
(Recall 1 Pa = 1 N/m^2)

In the eye of a hurricane, the ground-level atmospheric pressure may drop to 960 millibars = 96,000 Pa (about 5% reduction).

Gage pressure

Gage pressure is simply the pressure of a liquid or gas minus the atmospheric pressure at the place at the time.

Discovery of the atmospheric pressure

We walk around with the weight of the atmosphere on our shoulders, and since we are built to resist it and cannot get away from it, we tend not to notice it.

As a matter of fact, our ancestors did not even know about it. When Evangelista Torricelli (1608-1647) first demonstrated in 1643 in Pisa that there was such a thing as atmospheric pressure, he had some convincing to do.

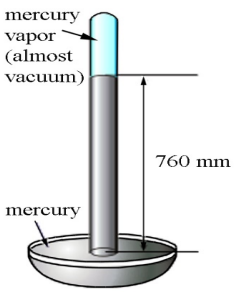


Figure 2-6. Sketch of Torricelli's first barometer.

Torricelli's experiment was rather elementary (Figure 2-5). It consisted in a long tube, approximately 1 m long, sealed at one end, filled with mercury, and inverted so that the open end dipped into a small bath of mercury. Atmospheric pressure pushing on the exposed mercury kept the mercury up in the tube, preventing it from falling down to the mercury level in the dish. Torricelli figured that the height at which the mercury was held inside the sealed tube could serve as a measure of the atmospheric pressure.

Precision in language

A barometer is a device to measure atmospheric pressure.
(From βαρυσ in Greek which means *heavy*.)

Thus, saying "**barometric pressure**" literally means the heavy pressure and makes little sense.
Yet it is often said when one actually means "**atmospheric pressure**."
Please don't make this mistake.

Pressure behind a dam



Figure 2-2. The Hoover Dam on the Colorado River at the border between Arizona and Nevada in the United States. It was completed in 1936 and rises about 220 meters (720 feet). The pressure against it at the bottom is about 21 times the atmospheric pressure.
Note the arch-shape to direct the tremendous force to the rock walls on both sides

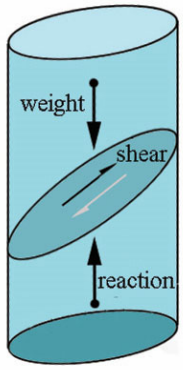
$$\rho\, g\, h = \left(1000\ kg/m^3\right)\left(9.81\ m/s^2\right)(220\ m) = 2,158,200\ \frac{N}{m^2} = 2,158.2\ kPa$$

$$\frac{\rho g h}{p_0} = \frac{2,158.2\ kPa}{101.325\ kPa} = 21.3$$

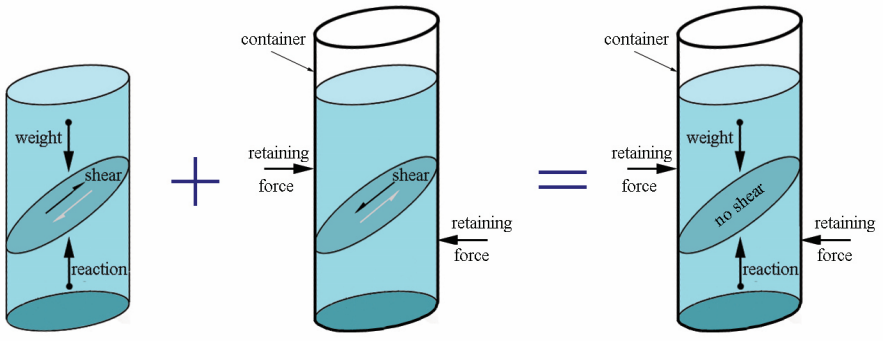
Internal stress: In a solid

Figure 2-3. Evidence of an oblique shear force in a broken column. The broken top portion of the column would slip down if it were not held by the surrounding brace. Likewise, a fluid cannot resist an oblique internal shear and needs to be held by the sides of a container.





Internal stress: In a contained fluid



The weight of the fluid above creates a shear along the oblique interface.

The side wall force creates an opposite shear along the oblique interface.

The sum of weight and side wall force combine to negate the shear along the oblique interface.

↓

No shear in the fluid causes no motion, and the fluid is at rest.

Pressure at a point has a unique value

One might think that since pressure is directed perpendicularly to the surface of the submerged object or side wall, the pressure must be a 3D vector. Not so! The pressure is actually the same in all directions and is just a value (a scalar). The perpendicular direction is a property of the surface, not of the fluid.

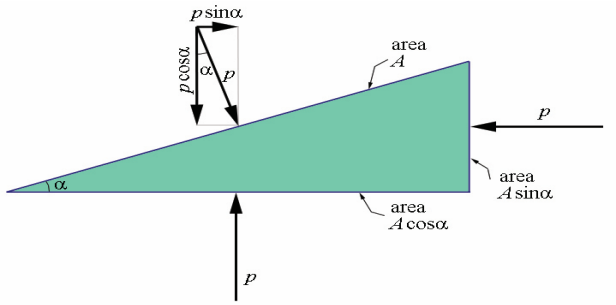


Figure 2-5. Pressure has a unique value regardless of the direction in which it is evaluated. The oblique pressure on top presses over a broader area, thus contributing to a larger force than the other pressure forces, but its projected components in the horizontal and vertical directions amount exactly to what the right and bottom pressure forces are as they press on smaller projected areas. Only with equal pressure in all directions can the wedge of fluid be in equilibrium.

Typical values of common pressures

	Typical pressure in Pa	Typical pressure in psi
Atmosphere at sea level	101,325	14.7
Automobile tire	$(1.9 – 2.2) \times 10^5$	28 – 32
Bicycle tire	6.9×10^5	100
Base of a 120 m (394 ft) dam	1.2×10^6	170
Propane tank	$(4.13 – 8.3) \times 10^5$	60 – 120
Birthday helium balloon	200,000	29
Basketball	55,160 – 58,600	8.0-8.5
Blood pressure	12,000 – 15,900	1.74 – 2.30

Pieces of trivia:

- Our blood pressure is about 4 times less than the air pressure in a basketball.
- The tire pressure in a bicycle is more than 3 times that in an automobile tire.

Manometers

A manometer is a device to measure the pressure in a fluid by means of a tube in which the fluid rises.

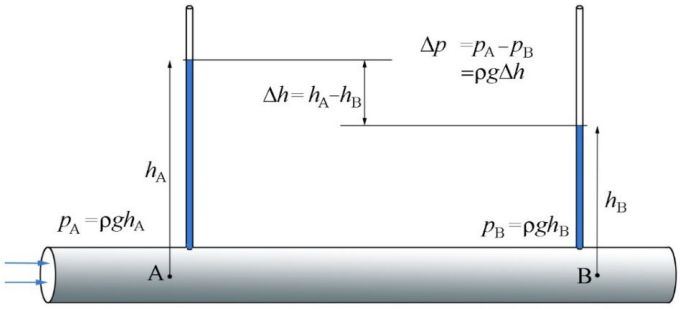


Figure 2-7. Two manometers placed along a conduit. While each manometer measures the pressure at its location, the difference between the two readings provides the pressure change down the flow, due to frictional loss.

Suppose now that the pressure inside the conduit is very large, which would necessitate impractically tall manometers, and that we are only interested in the pressure difference.

n that case, a U-tube configuration partially filled with a fluid heavier than that in the conduit is preferable

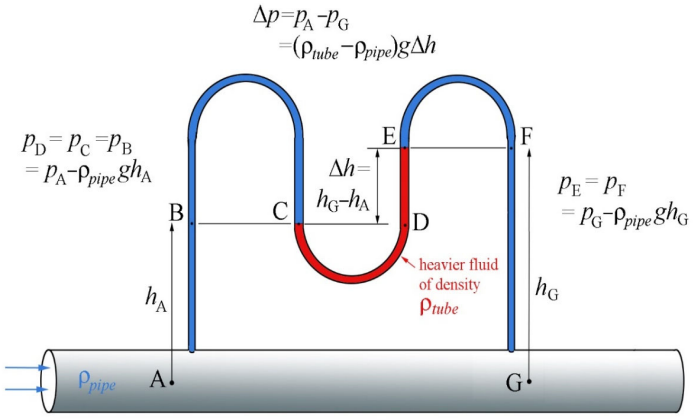


Figure 2-8. A U-tube configuration to measure the pressure difference between two points along a conduit from a single measurement.

Pressure gages

Manometers provide a nice visualization of pressures in a fluid, but they are impractical.

In practice today, people use pressure gages that provide an electric signal proportional to the pressure, usually the gage pressure. This permits convenient recording of the data, often over time in monitoring situations.
Example: Steam pressure in a boiler monitored in a control room.



Pressure gage – analog



Pressure gage – digital

Interesting application: The hydraulic piston

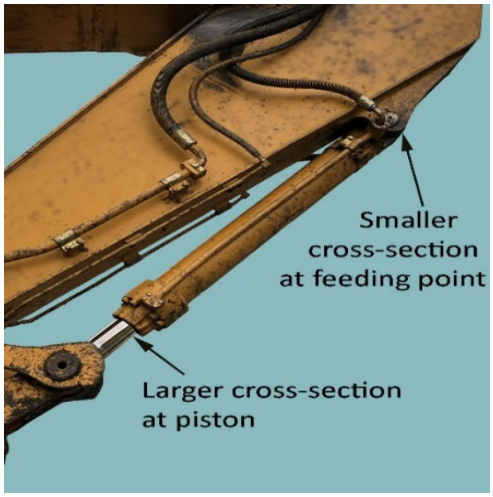


Figure 2-9. A hydraulic piston on an excavator arm.

Since pressure is a force per area,

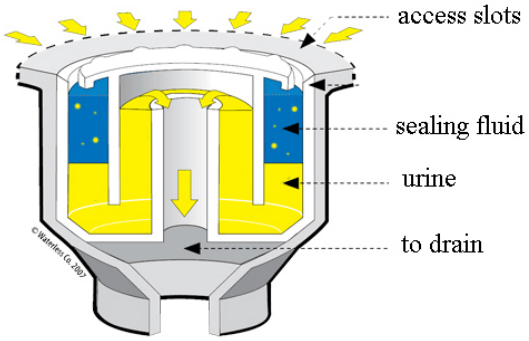
Force = Pressure x Area

A larger area causes a larger force from the same fluid pressure.

For example, the hydraulic system on the Caterpillar 325C LCR excavator operates at a gage pressure of 34.3 MPa. With a conduit inner diameter of 1.3 cm (0.50 in) and piston diameter of 14.0 cm (5.5 in), the force ratio is $(5.5/0.5)^2 = 121$, and the force that a piston can apply is $(34.3 \times 10^6\ N/m^2) \times (1.53 \times 10^{-2}\ m^2) = 5.26 \times 10^5$ N, corresponding to a weight of 53.6 metric tons (118,000 lbs).

Interesting application: The waterless urinal

The **objective** is to avoid the consumption of water, but urine smells and shouldn't be exposed to air. So, it must be covered by a fluid that won't be flushed with the urine.



The **solution** is to use a non-evaporating and non-smelling liquid, called sealing fluid, with a density lower than urine so that it floats on top of it. A typical oil will do.

The trick is to have a configuration that allows the urine to go down into a sewer pipe without losing the sealing fluid. This is accomplished by a down-up-down geometry as illustrated in the above sketch.

Elevation and pressure: Two forms of energy

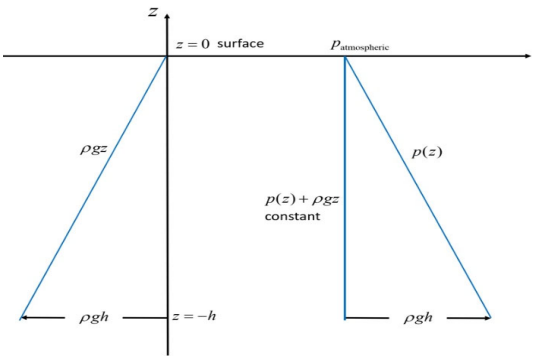


Figure 2-11. Pressure variation with depth in a motionless incompressible liquid.

As we go down with depth h, the pressure increases

$$p = \rho g h.$$

Since depth h corresponds to $z = -h$, it follows that

$p(z) + \rho g z =$ constant.

Now consider a volume V. Then,

Mass is $m = \rho V$ and

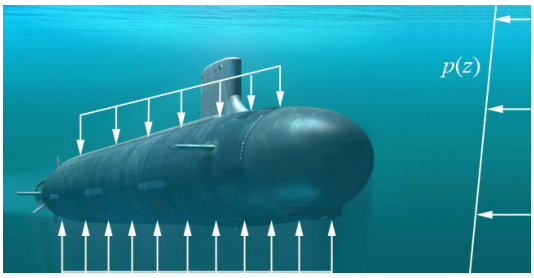
$pV + mgz =$ constant.

Potential energy

Pressure energy

Submerged bodies

The pressure distribution all around a submerged body adds up to a net force. The upward pressure on the underside is greater than the downward pressure on the top (Figure 2-12) simply because pressure increases with depth. There is thus a net upward force, which is called the *buoyancy force*.

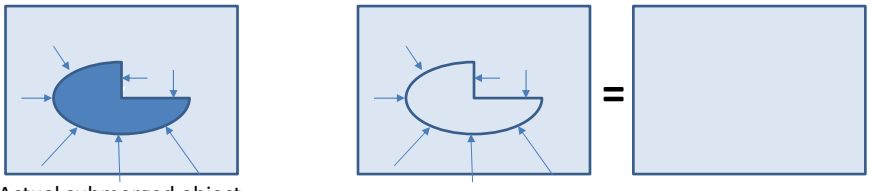


What is the amplitude of this force? At first glance, it may seem very complicated to obtain because of the possibly complex shape of the submerged body, but in fact it could hardly be simpler.

Figure 2-12. Pressure distribution around an underwater submarine. The water pressure on the underside exceeds that on the upper side, negating the weight of the submarine.
(*Photo credit*: Ultra Electronics Ocean Systems. Arrows added)

Imagine for a moment that we could draw through the fluid the exterior surface encompassing the submerged body and then magically remove the body from within and fill the cavity with fluid identical to the surrounding fluid.

What do we then have? Nothing special.
Just the same fluid everywhere, except that we have an imaginary enclosing surface



Actual submerged object

Submerged object replaced by same fluid as all around

Just plain fluid

The fluid within the enclosing surface, like any other parcel of fluid that we might define, is obviously in equilibrium with its weight perfectly balanced by the net pressure force exerted onto it by the surrounding fluid along the enclosing surface. Since this net pressure force is due to the weight of the surrounding fluid, it has nothing to do with whether the object within the enclosing surface is the original object or the substitute fluid. It follows that the net pressure force on a submerged body is equal to the weight of the fluid it replaces. Voilà!

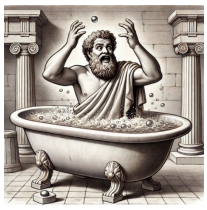
The net pressure force on a submerged body is equal to the weight of the fluid it replaces.

This is known as **Archimedes' Law of Buoyancy**.

In air, the buoyancy force is small and most often undetectable.

In water, by contrast, the buoyancy force is generally felt.
Bodies immersed in water feel lighter.
You feel lighter when you are in a swimming pool.

Archimedes' discovery



Archimedes (3rd Century BC)

Archimedes' task:
King Herio's new crown is suspected to be a mix of gold and silver. Can he provide evidence of the fraud?

When matched with a piece of pure gold of equal weight, the crown is balanced (left picture), but once dunked in the bath water, the crown happens to be lighter than the piece of gold (right picture).

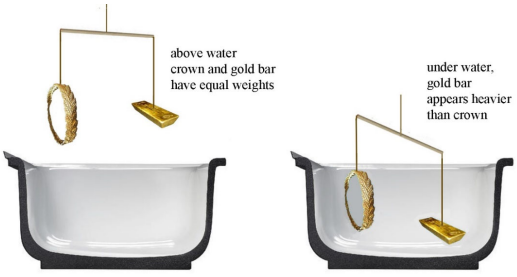
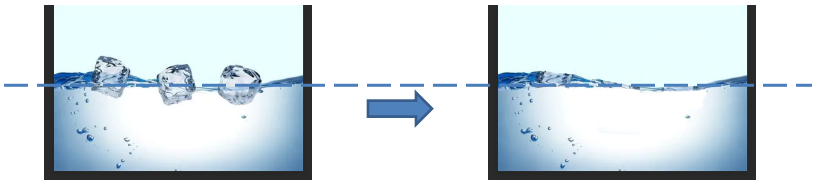


Figure 2-13. Archimedes' bathtub experiment to determine whether the king's crown was made of pure gold.

The conclusion is that the crown displaces more water than it would if it were made of pure gold.
It is therefore alloyed with a less dense metal.

An interesting consequence of Archimedes' Law of Buoyancy is that the melting of floating ice does not raise the water level, at least not over freshwater.
This is because while floating the ice displaces a volume of water of equal weight and once melted it simply occupies that same volume of water.



Floating Melted !

Icebergs do not contribute to sea level rise as they melt.



But they did create a sea level rise by displacement when the parent glacier moved from land onto the sea.

Floating bodies

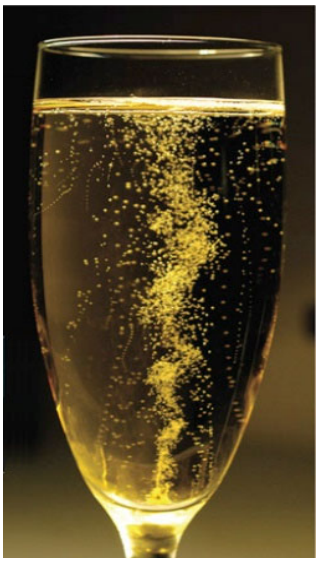


Figure 2-14. Rising bubbles of carbon dioxide in a glass of champagne.

The good and the bad…

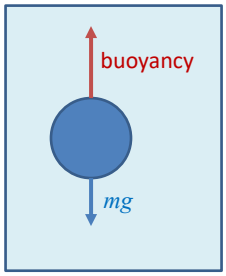


Figure 2-15. Empty plastic bottles floating on the sea.

Objects lighter than the surrounding fluid (most often water) rise until they can't anymore.

Bubbles burst at the surface.
Solid objects float at the surface.

Rule of floating bodies



According to Archimedes' Law of Buoyancy, a body lighter than the surrounding fluid is subject to an upward force greater than its own weight.
Thus, it will rise to the surface.

Once there, it will emerge from it, partially.

The level of partial emergence is such that the weight of the remaining displaced water (now reduced) equals the weight of the object.

At equal forces, the body no longer rises.

Floating objects displace
a volume of water the weight
of which equals their own weight.

Ship stability

G = Center of gravity of the ship
(may or may not be above water surface)
B = Center of buoyancy
= Center of gravity of the displaced water

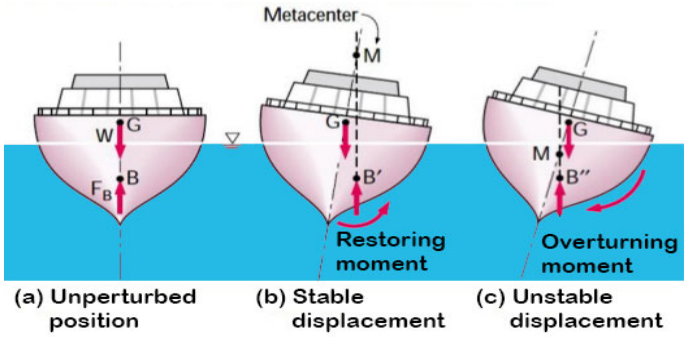


Figure 2-16. Cross-section of a floating ship indicating its center of gravity **G** and center of buoyancy **B**, in upright position [panel (a)] and two possible tipping positions. When the hull shape is configured so that the center of buoyancy **B** moves further out than the center of gravity **G** during tipping, the weight and buoyancy force create a moment in the restoring direction, and the ship is stable [panel (b)]. Otherwise [panel (c)], the tipping angle increases, and the ship may capsize depending on whether this force configuration persists or not with further tipping.

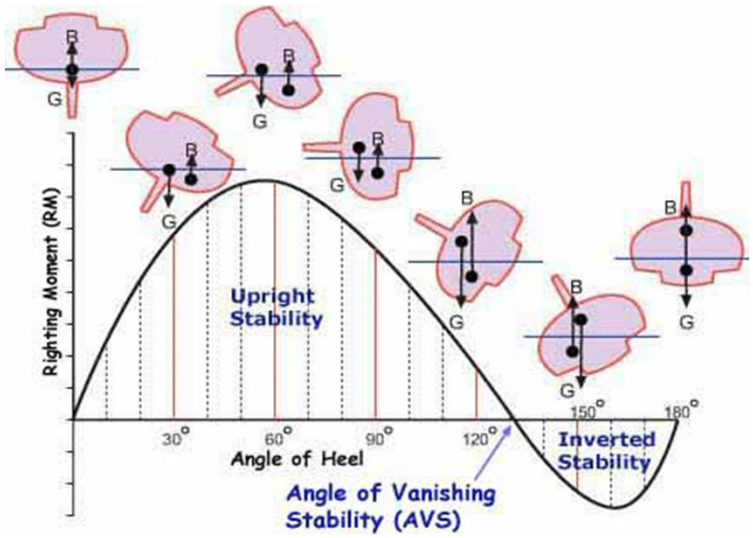


Figure 2-17. Stability of a hull under increasing tipping angle.

An 8-second video clip of a container ship tipping in a high wave:
https://www.youtube.com/watch?v=hNb81bdOkj4

A 31-second video clip of a ferry approaching the shore amidst waves in Hawaii:
https://www.facebook.com/NYPost/videos/ship-nearly-capsizes-runs-aground-amid-huge-swells-in-hawaii/1447533909616518/

How inflated tires support a vehicle

Just as water is strong enough to lift boats, one may be tempted to think that compressed air should be strong enough to lift vehicles.

In principle, a compressed air piston could lift any weight as long as the pressure is high enough. But, is it how an inflated tire works to keep a vehicle off the pavement? Actually not…

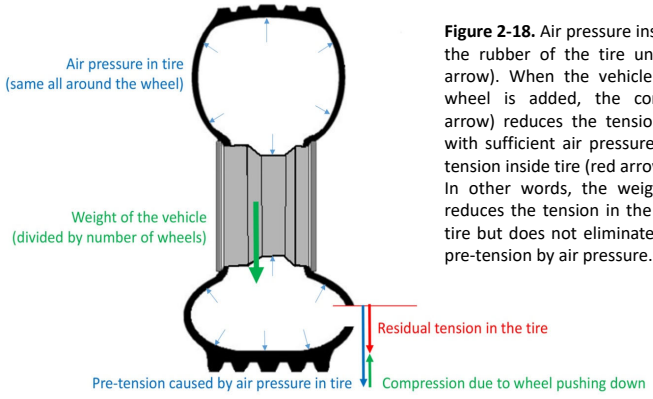


Figure 2-18. Air pressure inside the tire places the rubber of the tire under tension (blue arrow). When the vehicle weight onto the wheel is added, the compression (green arrow) reduces the tension in the tire, but with sufficient air pressure, there is residual tension inside tire (red arrow).
In other words, the weight of the vehicle reduces the tension in the lower part of the tire but does not eliminate it because of the pre-tension by air pressure.